B.Sc **Notes 11**

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Rolle's theorem

If a function f defined on $[a,b]$ is

(i) Continuous on $[a,b]$

(ii) derivable on $]a,b[$

(iii) $f(a) = f(b)$

then there exists at least one real number c between a and b $(a < c < b)$ such $f'(c) = 0$

Since the function is continuous on the closed interval $[a,b]$ it is bdd and attains its bd bounds. Thus if m and M are the infimum (g.l.b) and the supremum (l.u.b.) respectively of the function f then $\exists$ points c and d of $[a.b]$ such that

$$f(c) = m \text{ and } f(d) = M$$

There are two possibilities either $m = M$ or $m \neq M$

If $m = M$ then f is constant over $[a,b]$ and therefore its derivatives

$$f'(x) = 0 \ \forall \ x \in [a,b]$$

When $m \neq M$, both of these can't be equal to the same quantity

$f(a)$. At least one of these say m is different from $f(a)$ or $f(b)$ so that

$$f(c) = m \neq f(a) \Rightarrow c \neq a$$
$$f(c) = m \neq f(b) \Rightarrow c \neq b$$

This mean that c lies in the open interval $]a, b[$.
We shall now show that c is the point where $f'(c) = 0$

If $f(c) < 0$ then $\exists$ an interval $]c, x+\delta_1[$, $\delta_1 > 0$ for every point x of which $f(x) < f(c) = m$ which contradicts the fact that m is the infimum.
If $f'(c) > 0$ $\exists$ an interval $]c-\delta_2, c[$ $\delta_2 > 0$ for every points x of which $f(x) < f(c) = m$ which

is also a contradiction.
Hence the only possibility $f'(c) = 0$